Gravitat
force
Sir Isaac

Fundamental Particles

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1 Four Basic Forces **1**

Syllabus

Basic forces and classification of particles: The four basic forces, The force of gravity, the electromagnetic force, the week force and electroweak theory, the strong force. Conservations laws: Conservation laws and symmetries, conservation of energy and mass, conservation of linear momentum, conservation of angular momentum, conservation of electric charge, conservation of baryon and lepton numbers, conservation of strangeness, conservation of isospin and its components, the TCP theorem, conservation of parity. Quark model: The eightfold way, discovery of omega minus, the quark model, the confined quarks, experimental evidences for quark model, coloured quarks, quantum chromodynamics and gluons, Enough exercises.
Text: The particle Hunters : Yuval Ne'eman & Yoram kirsh Sections : 6.1-6.3, 7.1-7.11 and 9.1-9.8.
References: 1. Introductory nuclear Physics by Samuel S.M. Wong, Chapter 2 2. Introduction to Elementary Particles-David Griffiths.

1 Four Basic Forces

The four basic forces are identified as 1. gravitational, 2.electromagnetic, 3.Weak and 4.strong

Lecture:1-Gravitational force

Gravitation is quantitatively explained by Sir Isaac Newton in the 17^{th} century using his law of gravitation and second law of motion. The force on a body at a point having

inertial mass
gravitational mass
Newtons law of gravitation
Poisson's differential equation
Albert Einstein
general theory of relativity
extremum
geodesic
proper time

gravitational potential $\phi(r)$ is given by

$$m_I \frac{d^2\vec{r}}{dt^2} = -m_G \nabla\phi(\vec{r})$$

where m_I is the inertial mass which determines the acceleration under a force $(= F/a$) and m_G gravitational mass which determines the gravitational force in a field. Since gravitational acceleration is found to be independent of mass, $m_I = m_G$ and $\frac{d^2\vec{r}}{dt^2} = -\nabla\phi(r)$. Acceleration of a body at a point is determined by the gravitational field at that point.

For a continuous distribution of mass with a volume density $\rho(\vec{r'})$ at point $\vec{r'}$, the potential at a point $\hat{r}$ is given by

$$\phi(\vec{r}) = G\int \frac{\rho(\vec{r'})d^3r'}{|\vec{r}-\vec{r'}|}$$

where $G = 6.673\times 10^{-11} Nm^2kg^{-2}$ is the universal gravitational constant. This is the integral form of Newtons law of gravitation. It implies that gravitational field $\phi(r)$ is determined by the distribution of mass at other points $\vec{r'}$. Just like in electrostatics one can derive the differential form as

$$\nabla^2\phi(\vec{r}) = 4\pi G\rho(\vec{r})$$

which is a Poisson's differential equation.

This theory successfully explained the motion of planets around the sun, which represents motion with small velocities in weak gravitational fields. But Newton's law faced the following problems.

- It failed in the case of high velocities and strong gravitational fields.
- The transmission of gravitational force without a medium and with infinite velocity was not physically justifiable.
- Nearly the same conclusions can be obtained about the orbits of stars and planets if one chooses an inverse cube (r^{-3}) law or inverse 2.3 $(r^{-2.3})$-law.

These facts led Albert Einstein to propose his general theory of relativity. Einstein postulated a variational principle to obtain the equation of motion of matter in a gravitational field. *Freely falling bodies move along that curve for which proper time is an extremum.* Such a curve is called a geodesic. If τ is the proper time, $\delta\tau = 0$. If $x^\mu(\lambda)$ represents a path distinguished by the parameter λ and $\mu = 0,1,2,3$ That is,

$$\tau = \int \sqrt{\sum_{\mu,\nu} g_{\mu\nu}\frac{\partial x^\mu}{\partial\lambda}\frac{\partial x^\nu}{\partial\lambda}} d\lambda$$

where $g_{\mu\nu}$ is the metric of space-time. Using Einstein convention of summation over repeated indices

$$\delta \int \sqrt{g_{\mu\nu} \frac{\partial x^\mu}{\partial \lambda} \frac{\partial x^\nu}{\partial \lambda}} d\lambda = 0$$

. The Euler-Legrange equation for the extremum is then gives

$$\frac{d^2 x^\mu}{d\tau^2} = -\frac{1}{2} g^{\mu\nu} \left[\frac{\partial g_{\beta\nu}}{\partial x^\alpha} + \frac{\partial g_{\alpha\nu}}{\partial x^\beta} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu} \right] \frac{\partial x^\alpha}{\partial \tau} \frac{\partial x^\beta}{\partial \tau}$$

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where $\partial\tau^2 = g_{0\mu} dx^0 dx^\mu$. Comparing with $d^2\vec{r}/dt^2 = -\nabla\phi(\vec{r})$ one can see that in the relativistic equation $t \to \tau$ and field ϕ is replaced with metric $g^{\mu\nu}$ which determines structure of space-time. If metric g is diagonal with constant components ($g_{\mu\mu} = constant$), the spacetime is flat. If it is a constant but not diagonal, space time is not flat but has the same curvature everywhere. If $g_{\mu\nu} \equiv g_{\mu\nu}(x^\mu)$ are variables, curvature varies with events x^μ. Hence curvature of space-time controls the motion of bodies just like field determines acceleration in Newtonian gravity. The statement *'space-time curvature tells matter how to move'* is a consequence of it.

To estimate the effect of matter and energy on spacetime, Einstein put forward field equations which are the general relativistic analogue of the Poisson's equation. He planned his equations to satisfy the following requirements.

- It should pass all possible experimental tests.
- It should reduce to Newtonian gravity for weak fields.
- Locally it should satisfy special relativity.
- Gravitational fields can induce or modify other gravitational fields as field itself has energy density. Hence equation must be non-linear.
- Field equations must be consistent with energy-momentum conservation.

He postulated the relation between the geometry of space-time and the mass-energy-momentum as

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{c^4} T^{\mu\nu}$$

where $G^{\mu\nu}$ is the Einstein tensor, $R^{\mu\nu}$ is the Ricci tensor for curvature, $g^{\mu\nu}$ is the metric, $T^{\mu\nu}$ is the mass-energy-momentum tensor and R is the scalar curvature. The constant $(8\pi G/c^4) = 2.07 \times 10^{-43} kg^{-1} m^{-1} s^2$ is often called the Einstein's Gravitational constant. The components of T are

- $T^{00} \to$ energy density
- $T^{01}, T^{02}, T^{03} \to$ energy flux
- $T^{10}, T^{20}, T^{30} \to$ momentum density

momentum flux
perihelion
neutron stars
black holes
gravitational waves
graviton

- $T^{ij}, (i, j = 1, 2, 3) \rightarrow$ Stress or pressure or momentum flux

The Einstein tensor $G^{\mu\nu}$ is a non-linear function of the metric tensor, but is linear in the second partial derivatives of the metric. The second partial derivatives of the metric gives the curvature of spacetime. It is analogous to the Poisson's equation for Newtonian gravity except that the matter density $\rho(\vec{r})$ is now replaced with $T^{\mu\nu}$.

As a symmetric 2^{nd} rank tensor, the Einstein tensor has 10 independent components in a 4-dimensional space. The Einstein field equations are thus a set of 10 quasilinear second-order partial differential equations for the metric tensor. It can be shown that $\nabla_\mu G^{\mu\nu} = 0$ which ensures the conservation of the stress-energy-momentum tensor in curved spacetime as $\nabla_\mu T^{\mu\nu} = 0$ from field equation.

The change in the perihelion of the orbit of planet Mercury around the sun and the shift due to bending of light from stars as they pass through the gravitational field of other stars verified predictions of general relativity. It is shown that for particles moving with low velocities in weak gravitational fields, Einstein's equation reduces to Newton's law of gravitation. The description of super compact objects like neutron stars and black holes is impossible without general relativity. In general relativity, accelerated masses must emit gravitational waves. It is not observed yet as it is very weak.

Gravity is to be quantised for the following reasons.

- All other force fields like electromagnetic, weak and strong are already quantized . To integrate gravity with other fields, it must be quantized.

- In quantum theory of fields, force is exerted by the exchange of virtual bosons. The exchange boson for gravity is postulated to be a spin-2 graviton. The existence of graviton requires the quantization of gravity.

- We may combine the ideas of uncertainty and relativity in the following way.

$$\Delta x \geq \frac{\hbar}{2\Delta p} = \frac{\hbar}{2\Delta mc}$$

According to general relativity, the component g^{tt} of the metric near a point mass Δm is given by

$$g^{tt} = 1 - \frac{2G\Delta m}{c^2 \Delta x}$$

As $g^{tt} \geq 0$,

$$\Delta x \geq \frac{2G\Delta m}{c^2}$$

Multiplying the two equations for Δx one gets

$$\Delta x \geq \sqrt{\frac{G\hbar}{c^3}} = 1.616199 \times 10^{-35} m$$

This distance is called Planck's length. If gravity is quantised, existence of Planck's length will remove the divergences and singularities of relativity and quantum field theory.

But quantization leads to other problems two of which are given below.

- As gravity is space-time geometry, space-time will have to be quantised. Quantization of space-time makes its symmetries discrete.

- It is shown that if gravitational interaction is used to locate and measure the position and momentum, it can be more accurate than the respective uncertainty relations. This will violate the main principles of quantum theory.

Attempts are now in progress to quantize weak gravity fields by treating it as a perturbation to the Minkovsky space-time. This is called Ashtekar program for quantization. Witten program employs the idea of superstrings.

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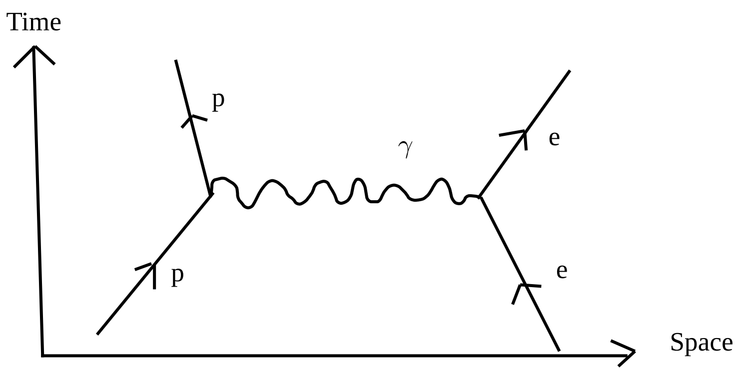
Lecture:2, Electromagnetic force



Electrostatic force between charges is given by Coulomb's law

$$\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$$

where r is the distance between charges q_1 and q_2, $\hat{r}$ a unit vector from q_1 to q_2 and $K = 9 \times 10^9 Nm^2C^{-2}$. As q's can be $+$ve or -ve, charges can attract (negative f) or repel (positive f). When such a charge moves, it is an electric current and a magnetic field is set up around it . The field $\vec{B}$ is given by Biot and Savart's law

$$d\vec{B} = \frac{\mu_0 i\vec{dl} \times \vec{r}}{r^2}$$

When a charge is accelerated, it emits energy in the form of electromagnetic radiation. The equations for electric and magnetic fields due to static and moving charges are

Maxwell's equations
Lorentz formula
Lorentz transformations
electromagnetic field tensor
Euler-Legrange equation
Equal-time commutation relations

given by Maxwell's equations

$$\begin{aligned} \nabla.\vec{B} &= 0 & (1) \\ \nabla \times \vec{E} &= \frac{\rho}{\epsilon_0} & (2) \\ \nabla \times \vec{E} + \frac{\partial \vec{B}}{dt} &= 0 & (3) \\ \nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{dt} \end{aligned}$$

Where $\vec{J}$ is the current density. The force on a charge q in an electromagnetic field is given by Lorentz formula

$$\vec{f} = q(\vec{E} + \vec{v} \times \vec{B})$$

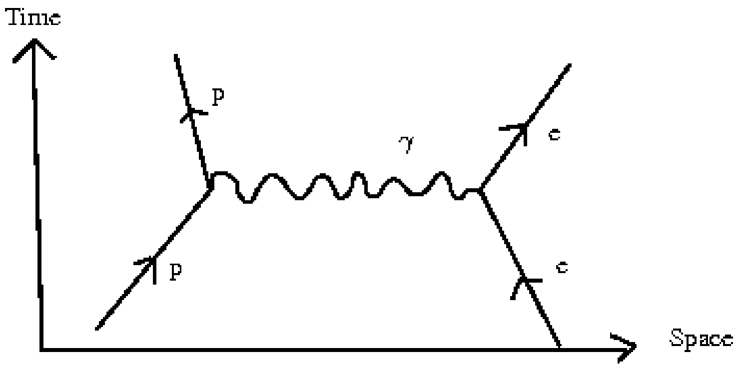
Maxwell's equations are invariant under Lorentz transformations. Hence relativistic equations for the fields can be expressed in terms of an electromagnetic field tensor F whose components are the three components of vector potential $\vec{A} = (A_1, A_2, A_3)$ and the scalar electric potential $\phi/c = A_4$

$$F^{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$$

As $F^{\mu\nu} = -F^{\nu\mu}$, the tensor is antisymmetric. Diagonal elements are zeros. Among the other twelve elements only six are independent. They can be identified as the three components each of electric field E and magnetic field B. The tensor is then given by [for Minkowski metric of signature (+,-,-,-)]

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

Electromagnetic field is the first one quantized successfully. It is done in canonical quantization by treating the potentials $A^\mu(\vec{x}, t)$ and conjugate momenta $\pi(\vec{x}, t)^\mu$ as operators. A suitable legrangian density is then chosen to derive the equation of motion using Euler-Legrange equation. Equal-time commutation relations between different components $[A^\mu, A^\nu]$,$[\pi^\mu, \pi^\nu]$ and $[A^\mu, \pi^\mu]$ are then postulated. It identifies photon with spin one as the exchange particle for force between electric charges.



Electromagnetic force appears

in quite a large class of natural phenomena. They act between atoms in molecules, causes friction, responsible for radiation etc. The fine structure constant $\alpha = e^2/2hc\epsilon_0 = 1/137$ is the measure of strength of electromagnetic interaction.

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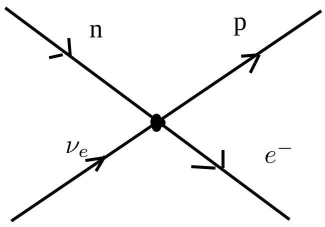
Lecture:3, The weak interaction

Weak interaction was first postulated when the strong nuclear forces cannot explain the decay of a neutron into a proton, the decay of pion into a muon and muon to an electron. These decays show the following characteristics.

- In most of these reactions a nutrino or antinutrino is emitted.
- The probability of their occurrence is low which shows this force is small in strength.
- The range of weak force is smaller than other nuclear forces ($< 10^{-17}m$).
- Parity is not conserved in these reactions.

One of the first theories of weak interaction is Fermi's theory of beta decay. In the early 1930's, inspired by the structure of electromagnetic interaction, Fermi postulated the beta decay $n \to p + e^- + \nu_e$ as the single-point four-fermion vector current interaction.



That is, interaction potential converts a neutron into a proton, electron and antineutrino instantaneously and at the same point in space. He wrote down the complete invariant amplitude for interaction as the product of hadronic weak current and leptonic weak current.

$$M = \frac{G_F}{\sqrt{2}} g^{\mu\nu} h_\mu i_\nu$$

where G_F is the Fermi constant, $g^{\mu\nu}$ the Lorentz metric, $h_\mu = \langle p| J_\mu^{(weak)} |n\rangle$ is hadronic weak current and $i_\nu = \langle e, \bar{\nu}_e| J_\nu^{(weak)} |0\rangle$ is leptonic weak current. Since the decay process takes place at a single point in space-time, the hadronic and the leptonic currents have common coupling constant and have identical transition operator $J^{(weak)}$. Fermi's theory also accounted for the dependence of weak force on the relative orientation of the spins of interacting particles. But the following differences with quantum electrodynamics were noted.

- In Fermi theory the total cross section for the decay process diverges with energy of the neutrino which is actually not true. Thus the first modification of Fermi

pion propagator
renormalisable
vector coupling hypothesis
parity violation
Vector-Axial vector

theory was made by introducing the intermediate vector bosons of spin 1, in analogy to photon in electrodynamics.

- The Current-current weak interaction changes electric charge of interacting particles. This leads to the fact that intermediate bosons must have electric charge. The charge of a particle does not change in electromagnetic interaction.
- Similar to pion propagator(Yukawa), intermediate boson must have mass that accounts for the short range of weak interaction while photon has zero rest mass.

The introduction of massive $W^{\pm}$ vector bosons as exchange particles leads one to include the propagator of the form

$$\frac{-g^{\mu\nu} + q^{\mu}q^{\nu}/M_W^2}{q^2 + M_W^2}$$

There are three vector bosons: $W^{\pm} \approx 80GeV, Z^0 \approx 91GeV$.
When the scattering of a neutrino by an antineutrino was considered, Fermi's theory predicted infinite probability. So the theory was not renormalisable. Another drawback of Fermi's theory was that it contained a large number of arbitrary parameters because the form of the weak force had been inferred directly from experiments. Fermi's vector coupling hypothesis allows the gauge bosons to possess only linear momentum and hence parity is conserved($\delta J = 0$). But parity violation in weak process is observed which indicates change of angular momentum.

In 1958 Marshak and Sudarsan put forward the Vector-Axial vector(V-A) theory of weak interaction. In axial vector coupling (angular momentum), the exchanged particle carries both linear momentum and angular momentum. We have $\delta J = 1$ which means that there is the violation of parity. Therefore, both the vector coupling and axial coupling parts contribute to the invariant amplitude of any weak process.

Electroweak theory

Similarities between electromagnetic and weak interaction made Salaam and Weinberg suggest a unified theory of weak and electromagnetic interaction called *electroweak theory*. Assuming a simplified picture of weak interaction by denoting the $W^{\pm}, Z^0$ couplings to quarks and leptons by the weak charge g_W, one would get (in natural units $h = c = 1$) from the matrix element for a single boson-exchange

$$f(q^2) = \frac{g_W^2}{q^2 + m^2}$$

where $q^2 = \Delta p^2 - \Delta E^2$, the 4-momentum transfer and m the average mass of $W^{\pm}, Z^0$ bosons. Comparing with e^2/q^2 for the electromagnetic scattering. For $q^2 << m^2$,

$$f(q^2) \approx \frac{g_W^2}{m^2}$$

is independent of q^2. It is similar to e^2/q^2 for the electromagnetic scattering. The interaction is pointlike. Fermi had postulated such an interaction of strength G_F between four fermions to describe nuclear beta-decay. Thus at low q^2

$$G_F = \frac{g_W^2}{m^2} \approx 10^{-5}(GeV)^{-2}$$

Weinberg and Salam proposed that the coupling g of the $W^{\pm}, Z^0$ bosons to leptons and quarks should be the same as that of the photon, i.e. $g_W = e$. The weak and electromagnetic interactions are thus unified with the same coupling. Then, from the measured value of G_F,

$$m \approx \frac{e}{\sqrt{G_F}} \approx \sqrt{\frac{4\pi\alpha}{G_F}} \approx 90GeV$$

which roughly agrees with the measured masses of W and Z particles.

Lecture:4, Strong Interaction

Strong force is a short-range force observed between nucleons. It is independent of their electric charge and provides stability to the nucleus as it causes nucleons to attract each other.

The first attempt to build a theory of force between nucleons was made by Hideki Yukawa. He assumed that to overcome the electrostatic repulsion between protons, the force must be attractive and of short range. The interaction potential is given the form

$$\phi(r) = -\frac{g_S e^{-mr}}{4\pi r}$$

where g_S represents the coupling between the nucleon and m a constant having dimension L^{-1}. The force acts by the exchange of mesons between nucleons. His theory is only partially successful because it offered no explanation for the following observations about the force between any pair of nucleons

- It may not act along the shortest path between the nucleons.
- It becomes stronger in the presence of other nucleons.
- It depends on the relative direction of spin of nucleons. Force between like spins is larger.
- It becomes repulsive at distances below $1.2 \times 10^{-15}m$. This is called hard-core repulsion.

These problems led to the formulation of a new gauge theory called the quantum chromodynamics. This theory assumes nucleons to be a collection of two types of particles called quarks. The force between quarks is responsible for the strong force between nucleons. These quarks move freely within the nucleon. Wilczek and others found that non-abelian gauge field has this property of asymptotic freedom. They introduced a new quantum number called *colour*. Quarks have *colour charge* and strong force is caused by the *colour*. As *colour* is conserved, there is also a *colour symmetry*. The exchange boson for the *colour force* is called a gluon with spin-1. Gluons themselves have colour and hence interact with other gluons. Asymptotic freedom is a consequence of this self-interaction.

The group of this gauge symmetry is SU(3). The gauge transformations involve three colour degrees of freedom called red, green and blue. The eight generators of SU(3) correspond to the eight massless gluons. As the energy of interaction decreases, the force among the quarks and gluons becomes too high for perturbation calculations. This leads to the phenomenon called quark confinement which means that quarks cannot be observed outside hadrons at low energies.

Lecture:5, Relative strength of the basic forces

The four fundamental forces have different ranges, depend on entirely different physical characteristics(eg. mass,charge,colour etc.) and is not having the same nature at all distances (strong force is attractive above 1.2 fm while it is repulsive below that distance.) There are no equivalent quantities to compare the strength of these forces. Hence in field theory, strength is measured using the following factors.

- The intrinsic mass of the exchange boson that transmits the force.
- The probability of exchange bosons being absorbed and emitted, measured as the reaction cross-section.
- The average lifetime of a particle undergoing an interaction.

Considering these factors, the strengths of the interactions are in the ratio

$$Strong : Electromagnetic : Weak : Gravitational :: 1 : 10^{-2} : 10^{-13} : 10^{-39}$$

Between two protons which are just in contact, all the four forces can act with strengths in the ratio

$$Strong : Electromagnetic : Weak : Gravitational :: 1 : 10^{-2} : 10^{-7} : 10^{-39}$$

Interaction	Strength	Range(m)	Exchange boson		
			Name	Rest mass	spin
Strong	1	10^{-15}	Muon	>100 MeV	0
Electromagnetic	10^{-2}	∞	Photon	0	1
Weak	10^{-13}	10^{-17}	$W^{\pm}, Z^0$	10 GeV	1
Gravitational	10^{-39}	∞	Graviton	0	2

Example of fundamental interactions: The formation of stars like the sun involves all the four interactions. During early stages of formation, interstellar dust and hydrogen condense under gravity. The gravitational potential energy appears as kinetic energy E of particles. The temperature T of this primordial ball increases as $T \propto E$. As condensation continues T increases. When $T \approx 10^7 K$, protons in Hydrogen collide with each other so that distance between some of them reduces to $10^{-17} m$. Weak interaction occurs and deuteron is produced.

$$p + p \rightarrow d + e^+ + \nu_e$$

When deuteron density reaches a threshold and temperature reaches $10^8 K$, strong interaction occur between colliding deuterons producing helium or tritium and releasing energy.

$$\begin{aligned} d + d &\rightarrow {}^3_2\mathbf{He}^* + n + 3.27 MeV \\ d + d &\rightarrow {}^3_1\mathbf{H}^* + p + 4.03 MeV \end{aligned} \tag{4}$$

The excited nuclei ${}^3_2\mathbf{He}^*, {}^3_1\mathbf{H}^*$ make transition to ground state by emission of a $\gamma -$ *photon* which is an electromagnetic interaction.

Lecture:6, The classification of particles

A large number of microscopic particles are detected already in high-energy particle collisions and cosmic rays. Different classification schemes for these particles are evolved. One of them employs the spin quantum number and the ability for strong interaction with other particles.

Particles with integral spin are called bosons. Energy distribution in boson systems follows Bose - Einstein statistics as they do not obey Pauli's exclusion principle. Particles with odd half-integral spin are called fermions. Energy distribution in fermion systems follows Fermi-Dirac statistics as they obey Pauli's exclusion principle. Some of the fermions and bosons engage in strong interaction while others abstain. Examples of such particles are given below.

	Strongly interacting	No strong interaction
Fermions	Baryons($p, n, \Lambda, \Sigma, \Xi, \Omega$...)	Leptons(e, μ, ν_e, ν_μ....)
Bosons	Mesons(K, π, η)	Exchange bosons($\gamma, W^\pm, Z^0$)

Fermions thus give mass to the universe while bosons determine how the particles combine to form the mass. The latest classification identifies only quarks and leptons as massive stable fermions. Gluons, $W^\pm, Z^0$ and photons are the exchange bosons in this scheme.

Conservation Laws

Every operation on a physical system which does not change its hamiltonian H is called a symmetry operation. There are three types of symmetries.

1. Continuous space-time symmetries like translation, rotation etc.
2. Discrete space-time symmetries like inversion.
3. Internal symmetries like isospin, strangeness, baryon number etc.

Continuous space-time symmetries

According to Noether's theorem, every continuous symmetry transformation of a field $\phi \rightarrow \phi + \delta\phi$ which leaves the action $\int \mathfrak{L} d^4x$ invariant implies the existence of a conserved '4-current' j. Here $\mathfrak{L}$ is the Legrangian density for the system. Conservation implies

$$\sum_{\mu} \partial_\mu j^\mu(x) \equiv \partial_\mu j^\mu(x) = 0$$

where $\mu = 0, 1, 2, 3$ and summation over repeated index is assumed(Einstein convention). The 'charge' density corresponding to (j_1, j_2, j_3) is given by $Q(t) = \int j^0(x) d^3x$ so that if $\int L dt$ is invariant, $dQ(t)/dt = 0$. There are three such global symmetries.

1. Space translation symmetry: $r_i \rightarrow r_i + \delta r_i$ so that $dp^i/dt = 0$ where p^i is the i^{th} component of linear momentum .
2. Time translation symmetry: $t \rightarrow t + \delta t$ so that $dE/dt = 0$ where E is the energy.
3. Rotation symmetry: $r_i \rightarrow \sum_j R_{ij} r_j$ with $RR^T = I$ so that $dL^i/dt = 0$ where L^i is the i^{th} component of angular momentum.

In quantum mechanics, the time evolution of an observable $\hat{A}$ is given by Heisenberg equation of motion

$$\frac{d\hat{A}}{dt} = i\hbar[\hat{H}, \hat{A}]$$

If $\hat{A}$ is conserved $d\hat{A}/dt = 0$ or $[\hat{H}, \hat{A}] = 0$. The observable commutes with the hamiltonian.
Problem: Prove the above conservation laws.

Lecture:7, Conservation of electric charge

Electric charge is neither created nor destroyed except in equal quantities of positive and negative charges. The total electric charge before and after any reaction must be equal. All electric charges are integral multiple of the basic unit charge $|e| = 1.602 \times 10^{-19} Coulomb$. The total charge of a system can always be expressed as an integer - positive, negative or zero. Since charge is quantized and conserved, an electron cannot decay spontaneously to a lighter particle as no negative charge of smaller mass exists. The symmetry associated with conservation of electric charge is the invariance of the Legrangian under the transformation of the vector potential $\vec{A}$ given by $\vec{B} = \nabla \times \vec{A}$

Conservation of Baryon and Lepton numbers

The non-occurrence of certain reactions even if mass-energy, linear and angular momenta, and electric charge are conserved shows the existence of other conservation laws. Two such conserved quantum numbers are baryon and lepton numbers.

Conservation of Baryon numbers: Proton-proton collisions at different energies have all possible exit channels. When proton energies suitable for the reaction $p + p \to \pi^+ + \pi^+ + \pi^o$ are set up, it is not found to occur. But a π^o meson is produced when a proton and antiproton scatter each other. When a large set of similar reactions involving baryons were analyzed, it is found that a new quantum number associated with baryons is conserved. This quantum number is given the name baryon number and assigned the following values to be consistent with observations. For baryons: $p, n, \Sigma, \Lambda, \Xi$ baryon no:=+1, for their antiparticles baryon no:=-1 and for leptons baryon no:=0. The baryon number of a nucleus is its mass number. Hence A denotes both the mass number and the baryon number of a nucleus.
The stability of proton is a consequence of the this baryon number conservation law because proton is the lightest baryon and hence it cannot decay to a baryon of smaller mass though positron is a lighter positive charge. Another consequence is that like electric charge, baryons are created and destroyed as baryon-antibaryon pairs . The symmetry responsible for baryon number conservation is not known so that the exactness of the law is doubtful.

Conservation of Lepton numbers: Leptons like electrons, muons and neutrinos are always created and destroyed in particle-anti-particle pairs. For example, a free neutron decays as $n \to p+e^- +\bar{\nu}_e$ while a proton in a nucleus may decay as $p \to n+e^+ +\nu_e$. This led to the postulate of a quantum number called lepton number and its conservation as (e^-, ν_e) are leptons while $(e^+, \bar{\nu}_e)$ are antileptons. The observation that a muon never decays to an electron implies that there is separate lepton number conservation law for electron leptons and muon leptons. Later it is discovered that the tau meson decays satisfying tau lepton number.The symmetry responsible for the different lepton number conservations are not known so that the law may be approximate.

The baryon number and lepton number conservation laws govern the creation and annihilation of all fermions. But bosons are created and destroyed without any limit as no such law exists for them.

Lecture:8, Conservation of strangeness

In proton-proton collisions of proper energies, kaons$(K^+, K^0, \bar{K^0}, K^-)$and hyperons $(\Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Omega^-)$ were created not as single particles but as pairs. Also it is found that though kaons are created by strong interaction in a time interval of $10^{-23}s$, they decay with a time constant of $10^{-10}s$ which indicates weak interaction.

Gellmann and Nishjima postulated that there is some common property to these particles. They called this property and its quantum number by the term *strangeness* represented as S. S can take only integer values $0, \pm 1, \pm 2, \pm 3, ...$ S is conserved in strong and electromagnetic interactions. If S is the strangeness of a particle, its antiparticle has strangeness $-S$. It is not conserved in weak interactions. S is zero for non-strange particles like nucleons, and mesons. They arbitrarily set the strangeness of Λ^o as -1. Assuming conservation in strong interactions, the following reaction gives strangeness of K^0 as $+1$.

$$p + \pi^- \to \Lambda^0 + K^0$$

$$S(K^0) = S(p) + S(\pi) - S(\Lambda^0) = 0 + 0 - (-1) = +1$$

Similar reactions gives the strangeness with increasing mass as follows.

Particle	K^+	K^0	$\bar{K}^0$	K^-	Λ^0	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-	Ω^-
Strangeness	$+1$	$+1$	-1	-1	-1	-1	-1	-1	-2	-2	-3

Strange particles decay by weak interaction.

$$\Xi^0 \to \Lambda^0 + \pi^0$$

$$\delta S = S(\Lambda^0) + S(\pi^0) - S(\Xi^0) = -1 + 0 - (-2) = +1$$

$$\Lambda^0 \to p + \pi^-$$

$$\delta S = S(p) + S(\pi^0) - S(\Lambda^0) = 0 + 0 - (-1) = +1$$

Every hadron possesses three properties. An electric charge Q, a baryon number (or charge)A, and a strangeness S. The quantum numbers associated with them are whole numbers $\pm n$, where $n = 0, 1, 2, 3, 4, ...$ .For anti-particle, these quantum numbers are opposite in sign to those of the particle. If (Q, A, S) represents a particle, its antiparticle will be $(-Q, -A, -S)$.

Isospin

It is found that the strong interaction is independent of electric charge. That is the pairs proton-proton, proton-neutron, neutron-neutron have the same strong force.

Two additional quantum numbers could be defined to explain this. The first of this pair called isospin I is conserved only in strong interactions, while the second called third component of isospin I_3 is conserved both in strong and electromagnetic interactions. Isospin is an internal symmetry whose symmetry group is SU(2). Isospin multiplets have nearly the same mass, such as the proton and neutron. This doublet of particles is said to have isospin 1/2, with projection (z-component) $+1/2$ for the proton and -1/2 for the neutron. The three pions $(\pi^{\pm}, \pi^0)$ compose a triplet, suggesting isospin 1. The z-components are $+1$ for the positive, 0 and -1 for the neutral and negative pions. Conversely, if there are n particles in a multiplet, the isospin of each of the particles in this multiplet is $I = (n-1)/2$. There is a relation between electric charge and isospin given by

$$q = e\left(I_z + \frac{S + A}{2}\right) = e\left(I_z + \frac{Y}{2}\right)$$

where $Y = S + A$ is called the hypercharge.

Conservation of Parity

The reflection at the origin of coordinate system ($\vec{r} \to -\vec{r}$) is called parity operation. Let $\hat{P}$ represents parity operator. Then $\hat{P}\vec{r} = -\vec{r}$. If the wave function of a system $\hat{P}\psi(\vec{r}) = \psi(-\vec{r}) = \psi(\vec{r})$ the parity of the system is positive or even. If $\hat{P}\psi(\vec{r}) = \psi(-\vec{r}) = -\psi(\vec{r})$ the parity is negative or odd. If parity remains the same after an operator acts on a wave function, parity is said to be conserved in that operation. For example, let $\hat{P}\left|\psi(\vec{r})\right\rangle = -\left|\psi(\vec{r})\right\rangle$ and $\hat{A}\left|\psi(\vec{r})\right\rangle = \left|\phi(\vec{r})\right\rangle$, parity is conserved if

$$\hat{P}\left|\phi(\vec{r})\right\rangle = -\left|\phi(\vec{r})\right\rangle$$

Parity is conserved in strong and electromagnetic interactions but need not be conserved in weak interactions.

Lecture:9, Charge Conjugation

This symmetry operation is associated with the interchange of particles and antiparticles. In the case of an electron, charge conjugation $\hat{C}$ performs reversal of sign of electric charge and the electromagnetic field. Hence invariance under charge conjugation demands the invariance of the legrangian density $\mathfrak{L}$, the electromagnetic four-current j_μ and potential A_μ..

$$\hat{C}\mathfrak{L}\hat{C}^{-1} = \mathfrak{L},\ \hat{C}j_\mu\hat{C}^{-1} = -j_\mu,\ \hat{C}A_\mu\hat{C}^{-1} = -A_\mu$$

If $|x\rangle$ represents a particle state, then its antiparticle is given by $|\bar{x}\rangle = \hat{C}\,|x\rangle$. If $|x\rangle$ is an electrically neutral state like K^0, n, Λ (non-hermitian fields), even then $\bar{K}^0, \bar{n}, \bar{\Lambda}$ are different because strangeness, baryon number etc changes under $\hat{C}$. The charge conjugation operator changes the sign of all the additive quantum numbers Q, B, L, I_3, but it does not change mass, energy, momentum or spin. Not all particles states are eigenstates of $\hat{C}$. For example, a proton is not the same as an antiproton. $\hat{C}\,|p\rangle = |-p\rangle \neq \pm\,|p\rangle$ as antiproton has different quantum numbers. But charges are eigenststes of charge operator $\hat{Q}$.

$$\hat{Q}\,|q\rangle = q\,|q\rangle\,,\ \hat{Q}\,|-q\rangle = q\,|-q\rangle$$

Time Reversal

Time reversal $\hat{T}$ refers to the discrete transformation $t \to t' = -t,\ \vec{r} \to \vec{r}' = \vec{r}$. In classical mechanics and electrodynamics, the basic equations are invariant under $\hat{T}$ since Newton's law of motion and Maxwell's equations are second-order differential equations in t. they are unaffected by the replacement $t \to -t$. Similarly, the quantum field $\psi(\vec{r}, t)$ transforms to $\psi(\vec{r}, -t)$. This leads only to a change of direction of conjugate momentum.. $\vec{p} \to -\vec{p},\ \vec{L} \to -\vec{L}$. Time reversal cannot be implemented using a linear unitary operator. The choice is to retain the unitarity of $\hat{T}$, but have $\hat{T}$ act on c-numbers as well as operators in the following way.

$$\hat{T}^{\dagger} = \hat{T}^{-1},\ \hat{T}(c - number) = (c - number)^{*}\hat{T}$$

where * represents complex conjugation. Since complex conjugation is non-linear, $\hat{T}$ is antilinear or antiunitary. Hence $\hat{T} = \hat{U}\hat{K}$, operator product of unitary and complex conjugation operators. The legrangian density for the field transform in such a way that the commutation relations are invariant.

$$\hat{T}\mathfrak{L}(\vec{r}, t)\hat{T}^{-1} = \mathfrak{L}(\vec{r}, -t)$$

In the case of electromagnetic currents j_{μ},

$$\hat{T}j_{\mu}(\vec{r}, t)\hat{T}^{-1} = j_{\mu}(\vec{r}, -t)$$

The currents are reversed while charges (j_0 is independent of time) are unchanged. $\hat{T}$ is a symmetry operator in this case

Table of symmetries

Physical quantity	Whether conserved in		
	Strong	EM	Weak
Electric charge	yes	yes	yes
Baryon number	yes	yes	yes
Electron lepton number	yes	yes	yes
Muon lepton number	yes	yes	yes
Taon lepton number	yes	yes	yes
Colour	yes	yes	yes
Strangeness	yes	yes	No
Flavour	yes	yes	no
Isospin(I)	yes	No	No
Third component of isospin(T_3)	yes	yes	No
Parity(P)	yes	yes	No
Charge conjugation(C)	yes	yes	No
CPT	yes	yes	yes

Lecture:10, PCT-theorem

PCT-theorem or Luders-Pauli theorem states that all interactions in nature, all the force laws, are invariant on being subjected to the combined action of reflection of the coordinate system through the origin called parity (P), particle-antiparticle interchange called charge conjugation(C), and reversal of time(T). The operations may be performed in any order. If H is the hamiltonian, $[H, PCT] = 0$. PCT theorem leads to the following conclusions.

1. A particle and the corresponding antiparticle have equal masses and equal lifetimes.

2. The electric charges of a particle and the corresponding antiparticle differ only in sign, as do the magnetic moments.

3. The interaction of a particle and of the antiparticle with a gravitational field is identical.

4. In cases where the interaction of particles in the final state is negligible, the energy spectra and angular distributions of the decay products are the same for particles and anti-particles, and the projections of the spins are of opposite sign.

As a test of PCT-theorem the following ratio's were determined experimentally.

$$\frac{M(K^0) - M(\bar{K}^0)}{M(K^0) + M(\bar{K}^0)} < 10^{-19}$$

$$\frac{M(e^+) - M(e^-)}{M(e^+) + M(e^-)} < 4 \times 10^{-8}$$

$$\frac{Q(p) - Q(\bar{p})}{|e|} < 2 \times 10^{-5}$$

$$\frac{\tau(\mu^+) - \tau(\mu^-)}{\tau(\mu^+) + \tau(\mu^-} < 10^{-4}$$

where M, Q and τ refers to mass, electric charge and mean life.

Lecture:11, The SU(3) symmetry

The charge independence of strong interactions led to the concept of isospin symmetry. It refers to the invariance under rotation in isospin space. The group of transformations, which generate these rotations, is the SU(2) group. In the case of nucleons, this is the fundamental 2-D representation of SU(2) defined by the proton and neutron. A nucleon state is a mixture of proton and neutron states. If g is an element of SU(2) , then

$$g\begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} p^* \\ n^* \end{pmatrix}$$

Similarly pions (π^+, π^0, π^-) defines a 3-D isospin space and the four Δ-particles a 4-D space. The particles correspond to the 3-D and 4-D representations of the SU(2) group.

When the conservation of strangeness is observed in strong interactions, it became clear that the symmetry group governing it is larger than SU(2). The natural choice is the SU(3) group This group generates many irreducible representations designated by two parameters (p, q). The possible dimensions are $d = (1 + p)(1 + q)(2 + p + q)/2$

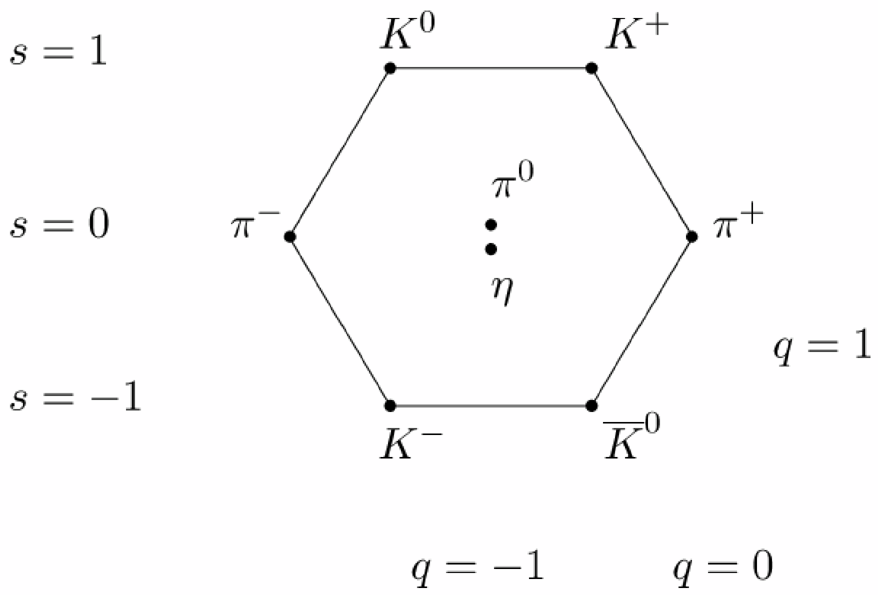


Figure 1: Zero-spin Meson Octect

where $p, q = 0, 1, 2, 3...$ Therefore, the dimensions are $(0, 0) = 1,\ (1, 0) = 3,\ (0, 1) = \bar{3},\ (1, 1) = 8,\ (2, 0) = 6,\ (3, 0) = 10,\ (2, 2) = 27...$ each of which is a well-defined quantum number pattern. The assignment of hadron multiplets to the 8-D and 10-D representations of SU(3) was successful and predicted many new particles. But the absence of particles in the fundamental 3-D representation of SU(3)namely $(1, 0) = 3$ and $(0, 1) = \bar{3}$ remained unexplained.

The Eight Fold Way

By 1960, hundreds of hadrons were detected and their properties studied. It was found that the heavier the hadron, the smaller its mean life. Attempts were made by Gellmann and Yuval Nee'man to find some symmetry scheme to classify them. The scheme they created is called the 'Eight-fold Way classification'. If we take three flavours of quarks, then the quarks lie in the fundamental representation, *3* (called the triplet) of flavour $SU(3)$. The anti-quarks lie in the complex conjugate representation $\bar{3}$. The nine states (nonet) made out of a pair can be decomposed into the trivial representation, 1 (called the singlet), and the adjoint representation, 8 (called the octet). The notation for this decomposition is $\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$. The spin-1/2 baryons and the spin-0 mesons were arranged along the corners and center of a regular hexagon in the $I_z - S$ plane. It was found that if charge is the same for particles falling along some line, it is so for lines parallel to it too. The success of this scheme includes the following predictions, which were later confirmed.

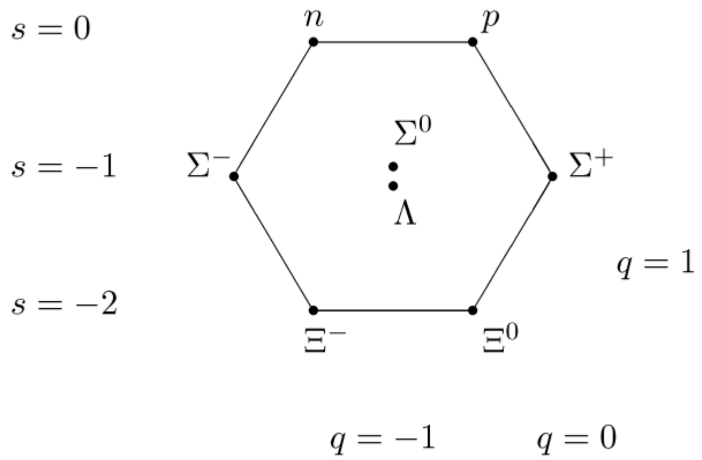


Figure 2: Half-spin Baryon Octet

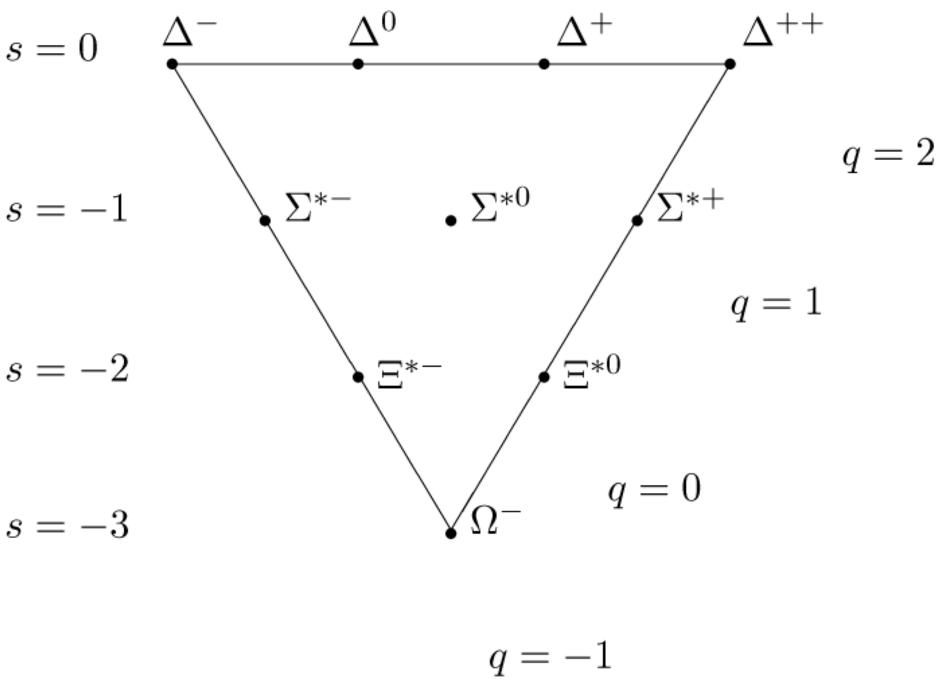


Figure 3: 3/2-spin Baryon-decuplet

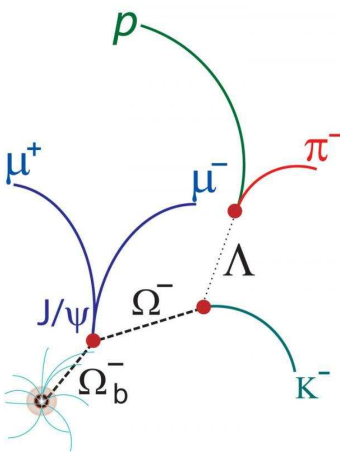


Figure 4: Decay of Ω^-

- Mesons with mass 570MeV exist,
- Ξ baryons would have spin 1/2
- Nucleon resonances with positive strangeness could not exist.
- A new doublet of Ξ^* resonances with strangeness s=2 and a singlet Ω^- with s= -3 must exist.

Lecture:12, Discovery of Omega minus particle

Analysing 100 trillion proton-antiproton collisions produced at Fermilab, 18 incidents were found in which the emerging particles revealed the formation of the Ω_b^- particle. The Ω_b^- disintegrates into two intermediate particles called J/Ψ and Ω^-. The J/Ψ then promptly decays into a pair of muons. The Ω^- baryon decays into the unstable particle Λ baryon along with a long-lived particle called kaon (K). The Λ baryon, which has no electric charge, also can travel several centimeters prior to decaying into a proton and a pion. Theorists predicted the mass of the Ω_b^- baryon to be in the range of $5.9 to 6.1 GeV/c^2$. The measured mass is $6.165 \pm 0.016 GeV/c^2$. The particle has the same electric charge as an electron and has spin J=1/2.

Quarks

In 1964 Gellmann and Zweig pointed out that the representations of SU(3) may be chosen by assuming them to be generated by just two combinations of the fundamental 3-D representation. He made the following assumptions.

1. The particles belonging to this representation are named up(u), down(d) and strange(s) quarks.

2. All quarks have their antiquarks also.
3. They are point-like.
4. They could be described by Dirac Equation.
5. They are half- spin fermions.
6. They carry fractional charge.
7. A quark antiquark pair forms a meson while baryons contain three quarks. No other combination is possible.
8. The different energy states of quarks are widely separated in mass so that combinations of different states are different particles.
9. $q \otimes \bar{q} = 3 \otimes 3 = 1 \oplus 8$. A singlet and an octect states are thus possible for mesons. For baryons $q \otimes q \otimes q = 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$. Hence singlet, octect and decuplet states are possible.
10. Only particles whose properties do match any one of the above two combinations could exist. eg. Baryons with s=0,q=-2 and mesons with q=+2 and s=-3 must not exist.

This quark model faced two main difficulties. As quarks have the smallest electric charge, they must be stable for charge conservation. But quarks are not detected experimentally. Quarks are fermions and must obey Pauli's exclusion principle. The simultaneous presence of identical quarks in baryons violates exclusion principle. The former problem is tackled with the idea of quark confinement. The latter is overcome by defining a new quantum number called 'colour'.

Properties of quarks:

quark	Isospin(I)	I_3	Strangeness(S)	Baryon no:(A)	Electric charge(Q)
u	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{2}{3}$
d	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	-$\frac{1}{3}$
s	0	0	-1	$\frac{1}{3}$	$-\frac{1}{3}$
$\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{3}$	-$\frac{2}{3}$
$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{3}$	$+\frac{1}{3}$
$\bar{s}$	0	0	$+1$	$-\frac{1}{3}$	$+\frac{1}{3}$

Lecture:13, Quark Structure of baryons and mesons

Baryons and mesons are colourless. A quark and antiquark of one colour and its anti-colour form a meson. Three quarks of different colours form a baryon. From the three types of quarks- up,down and strange- and their antiquarks, one can form 8 mesons as follows.

Combination	I_3	Y=A+S	$Q = I_3 + Y/2$	Particle
$u\bar{d}$	+1	0	+1	π^+
$u\bar{s}$	1/2	+1	+1	K^+
$d\bar{u}$	-1	0	-1	π^-
$d\bar{s}$	-1/2	+1	0	K^0
$s\bar{u}$	-1/2	-1	0	K^-
$s\bar{d}$	1/2	-1	0	$\bar{K}^0$
$(u\bar{u} - d\bar{d})/\sqrt{2!}$	0	0	0	π^0
$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{3!}$	0	0	0	η^0

Quark Confinement

The failure to detect free quarks led to the idea of Quark Confinement. The deep inelastic scattering of a proton by proton, electron, neutrino etc. has shown that most of them passed through the scatterer without deviation while a few deviated sharply and still fewer bounced back. This shows that proton has structure and pattern of deviation shows three point charges within it. This is indirect evidence of the existence of quarks. The inter-quark forces are zero within the proton. If an incident particle has enough energy to release a quark from a proton, its energy will be converted into a quark-antiquark pair. This quark will replace the lost quark of the proton and the antiquark will combine with the extracted quark to form a meson. So any scattering will not produce a free quark.

The force between quarks is attractive and is expected to increase with separation. $F = \infty$ as $r \to \infty$. So a perfectly free quark is not a reality.

Charm quark

In November 1974, C.C.Ting and B.Ritcher discovered a new meson called ' ψ ' with 3 times the proton mass and a mean life of 10^{-20}s. As no set of $q\bar{q}$ can have such masses, a fourth quark called 'charm' was predicted with its properties. $\psi = c\bar{c}$. Charmed mesons (D^0, D^+) and baryons(Λ^+, Σ^{++}) were discovered by 1976.

The discovery of τ-lepton and the τ-neutrino led to a search for new quarks which resulted in the discovery of bottom quark (beauty) and top (truth) quark. These quarks are heavier than u, d, s and c-quarks.

The present picture of elementary particle physics is this
6 spin-half leptons : $e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$ and 6 anti-leptons
6 spin-half Quarks : u, d, s, c, b, t and 6 anti-quarks
12 spin-one Exchange bosons: Photon(1), gluon(8), $W^{\pm}(2)$ and Z^0-particle(1)

Experimental evidences for quark model

The existance of an internal in the proton was provided by the following experiments.

1. The scattering of 20 GeV electrons by stationary proton targets, it was able to penetrate into the volume of proton and probe its inner structure. It was found that most of the electrons that passed through the target underwent only small deviations from their original path. But the number of large-angle deviations was significant. This indicates that the charge and mass are not uniformly distributed within the proton, but there are point-like centres. They may be identified as quarks.

2. Consider a $40GeV - \nu_\mu$ interaction with a proton. A large number of particles including hadrons may be produced in a single reaction. It is found that a number of hadrons were emitted in a narrow beam at a large angle with the incident direction. It may be interpreted as a collision between the neutrino and a quark inside the proton. As a result new quarks and anti-quarks are created which constitute the observed beam of hadrons, all moving in nearly the same direction.

3. In collisions between two beams of protons at a total energy of up to $63GeV$, part of the energy is converted into a number of particles. A few of them are ejected in directions perpendicular to the beam direction. Such beams indicate the presence of charge-centres inside the proton. The jet is the result of a head-on collision between a quarks in the protons.

4. The study of very high-energy $e^- - e^+$ annihilation events, the observed products are a number of hadrons, concentrated in two narrow jets moving in opposite directions. It was concluded that the initial products of the annihilation are a quark and antiquark pair. As they move apart, they form a set of new quark-antiquark pairs. They combine to form the two hadron beams. They move in opposite directions to conserve the momentum.

color force
gluons
asymptotic freedom

5. When protons were bombarded with 10 GeV pions, pairs of muons, μ^+, μ^- were found. It turned out that when the bombarding pions were negative, four times as many muon pairs were formed than when the pions were positive. It was also found that in proton-proton collisions at the same energies, the production of muon pairs was a much rarer event. The muons are produced by annihilation of a quark in the proton and an anti-quark in the pion. Since protons contain no anti-quarks, such annihilations cannot occur in proton-proton collisions. Therefore formation of muon pairs in $p-p$ process is very rare.

Lecture:14, Coloured Quarks

Quarks are fermions and must obey Pauli's exclusion principle. The simultaneous presence of identical quarks in baryons violates this principle. To overcome this difficulty O.W.Greenberg suggested that in addition to the three flavours-up,down and strange-the quarks are characterized by a new property which he called 'colour'. Eachflavour comes in three colours *red, green* and *blue.* If 'redness' of a quark is $+1$, that of its antiquark is -1. The combination of a quark and antiquark of the same colour will have no colour. The three colours in equal ratio is white and hence colourless. In baryons, the three quarks have different colour. Pauli's exclusion principle is not violated and baryons become colourless. Mesons are made up of a quark and an antiquark of the same color so that it is colorless. It is now stated as a principle. *All naturally occurring stable particles are colourless.* This explains the absence of stable particles having even number of quarks or antiquarks.

Color Force

The force between quarks is called the color force. Since quarks make up the baryons, and the strong interaction takes place between baryons, the color force is the source of the strong interaction, or that the strong interaction is a residual color force which extends beyond the proton or neutron to bind them together in a nucleus.

Inside a baryon the color force show unusual properties which are not seen in the strong interaction between nucleons. The color force does not decrease with distance and is responsible for the confinement of quarks. The color force involves the exchange of gluons and is so strong that the quark-antiquark pair production energy is reached before quarks can be separated. Gluons carry a color and anticolor so that inter-quark interactions are invariant under color interchange. The gluons interact among themselves due to their colored state. Out of the nine possible combinations of color and anticolor, only 8 gluons are distinct. The members of this octet are:$(r\bar{b}), (r\bar{g}), (b\bar{r}), (b\bar{g}), (g\bar{b}), (g\bar{r}), \frac{(r\bar{r}-b\bar{b})}{\sqrt{2}}, \frac{(r\bar{r}+b\bar{b}-2g\bar{g})}{\sqrt{6}}$. Another property of the color force is that it exerts little force at short distances. The quarks are free particles within the boundary of the nucleon. This behavior of the gluon interaction between quarks is called the *asymptotic freedom* . They experience the strong confining force when inter-quark distance exeeds nucleon diameter.

Quantum chromodynamics (QCD)

Higgs bos

The force between coloured particles like quarks and gluons is called chromodynamic force. The theory of colour interactions is called quantum chromodynamics (QCD). The theory assumes that the chromodynamic force is transmitted by eight massless and chargeless spin- 1 and coloured particles called gluons. Each gluon carries a colour and an anti-colour and gluons exert chromodynamic forces on each other.The colour of the quark may change during emission or absorption of a gluon. When a red d-quark emits a gluon carrying the red and anti-green, it becomes a green d-quark. A green u- quark absorbing this gluon will become a red u-quark. There are six colour-changing gluons: $(r\bar{b}), (r\bar{g}), (b\bar{r}), (b\bar{g}), (g\bar{b}), (g\bar{r})$ and two colour-preserving gluons:$\frac{(r\bar{r}-b\bar{b})}{\sqrt{2}}, \frac{(r\bar{r}+b\bar{b}-2g\bar{g})}{\sqrt{6}}$. The transmission of the chromodynamic force by gluon-exchange causes the quarks in a hadron to change colours continuously. Since hadrons are colourless, this change of colours cannot be detected. Like an accelerated electric charge emitting photons, an accelerated quark is expected to emit gluons.

Lecture:15, Standard Model

The Standard Model of particle physics describes the strong, electromagnetic and weak forces, as well as the fundamental particles that make up matter. It is a quantum field theory consistent with both quantum mechanics and special relativity. It assumes that all interactions are identical at a distance of $10^{-19}m$. The apparent difference is just a 'long-distace' effect. To date, almost all experimental tests of the forces described by the Standard Model have agreed with its predictions. It is not a complete theory of fundamental interactions, because it does not describe gravity.

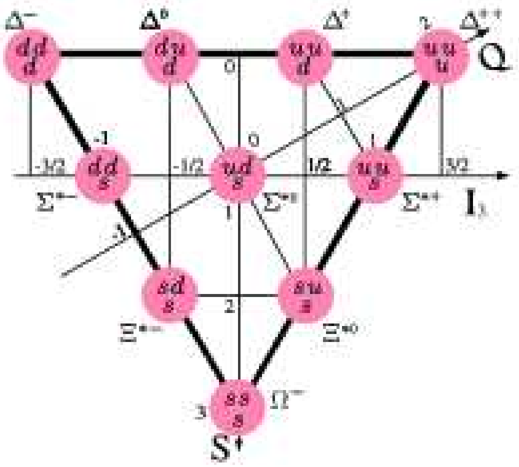
The Standard Model contains both spin-1/2 fermions and bosons of spin 1 and 2 as fundamental particles. Fermions are particles of matter while bosons transmit forces. The theory of the electroweak interaction is combined with the theory of quantum chromodynamics. These theories are gauge field theories, meaning that they describe the forces between fermions by coupling them to bosons, which mediate the forces. The Lagrangean of these bosons is invariant under gauge transformation, so these mediating bosons are referred to as gauge bosons. The bosons in the Standard Model are:

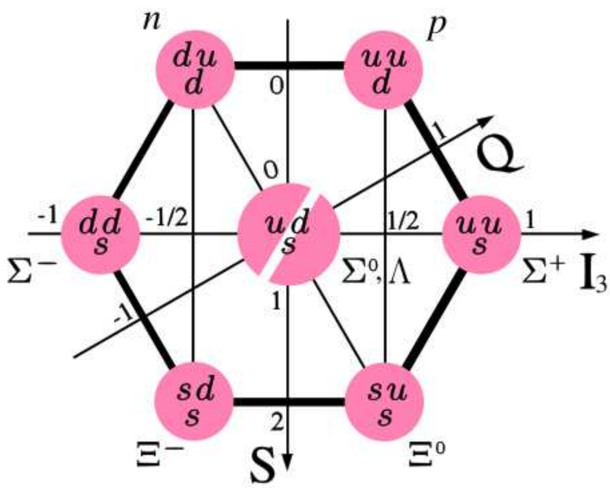
- Photons, which mediate the electromagnetic interaction.
- The set of 3 bosons (W^+, W^-, Z^0) mediate the weak nuclear force
- Eight species of gluons mediate the strong nuclear force. Six of these gluons are labeled as pairs of "colors" and "anti-colors" (for example, a gluon can carry "red" and "anti-green".) The other two species are a more complicated mix of colors and anti-colors.
- The Higgs boson, which induce spontaneous symmetry breaking of the gauge groups and is responsible for the existence of inertial mass.

It turns out that the gauge transformations of gauge bosons can be exactly described using a unitary group called a "gauge group". The gauge group of the strong interaction is SU(3), and the gauge group of the electroweak interaction is $SU(2)\times U(1)$. Therefore, the Standard Model is often referred to as $SU(3)\times SU(2)\times U(1)$. The Higgs boson is the only boson in the theory, which is not a gauge boson; it has a special status in the theory, and has been the subject of some controversy. It is discovered in May,201 in the Large Hadron Collider during high-energy $p-p$ scattering. Most of the results of standard model are verified experimentally. The culmination of its agreement with experiment occured recently when the gyromagnetic ratio of electron is measured. The theoretical value is $g_e^{Th} = 2.00231930431$ while the experimental value is $g_e^{Exp} = 2.00231930437$.

Limitations of standard model:

1. The gravitational interaction is not included in this model.
2. Neutrinos are assumed massless. But there is experimental evidence for their very small but non-zero mass.
3. There are about 17 parameters in the model whose values are assigned arbitrarily. The source of most of these values are still unknown.
4. The occurrence of 6 fundamental leptons and exactly the same number of quarks as the basic building blocks is not explained.
5. The existence of dark matter and the matter-antimatter asymmetry in mass is not accounted for.





Tutorial Problems: 6hours

1. Are the following particle interactions allowed by the conservation rules? If so, state which force is involved. $(i)\mu \to e + \nu_\mu + \nu_e$, $(ii)\Lambda \to \pi^+ + \pi^-$, $(iii)\nu_e + n \to p + e^-$, $(iv)\pi^0 \to \tau^+ + \tau^-$, $(v)e^+ + e^- \to \mu^+ + \mu^-$

2. Indicate, with an explanation, whether the following interactions proceed through the strong, electromagnetic or weak interactions, or whether they do not occur. $(i)\pi^- \to \mu^- + \nu_\mu$, $(ii)\tau^- \to \mu^- + \nu_\tau$, $(iii)\Sigma_0 \to \Lambda + \gamma$, $(iv)p \to n + e + + \nu_e$, $(v)\pi^- + p \to \pi_0 + \Sigma_0$, $(vi)\pi^- + p \to K_0 + \Sigma_0$, $(vii)e^+ + e^- \to \mu^+ + \mu^- (viii)\Xi^- \to \Sigma^- + \pi^0$, $(ix)\tau^- \to e^- + \nu_e + \nu_\tau$, $(x)\tau^+ \to \mu^+ + \gamma (xi)\mu^+ + \mu^- \to \tau^+ + \tau^-$, $(xii)p \to e^+ + \pi^0$, $(xiii)\pi^0 \to \gamma + \gamma$, $(xiv)\pi^- + p \to K^+ + \Sigma^- (xv)\pi^- + p \to K^- + \Sigma^+$

3. Consider the decay of K^0 meson of momentum P_0 into π_+ and π^- of momenta p^+ and p^- in opposite directions such that $p^+ = 2p^-$. Determine p^0 . $[M(K_0) = 498MeV/c^2; M(\pi^\pm = 140MeV/c^2]$

4. The baryon Ω^- has a mass $1,672MeV/c^2$ and strangeness $s = -3$. Which of the following decay modes are possible? $(a)\Omega^- \to \Xi^- + \pi^0 [m(\Xi^-) = 1,321MeV/c^2, S(\Xi^-) = -2, m(\pi^0) = 135MeV/c^2]$, $(b)\Omega^- \to \Sigma^0 + \pi^- [m(\Sigma^0) = 1,192MeV/c^2, S(\Sigma^0) = -1, m(\pi^-) = 139MeV/c^2]$, $(c)\Omega^- \to \Lambda^0 + K^- (m_\Lambda = 1,115MeV/c^2, S_\Lambda = -1, m_{K^-} = 494MeV/c^2, S_{K^-} = -1$, $(d)\Omega^- \to n + K^- + K^0 (m_{K^0} = 498MeV/c^2, S_{K^0} = -1)$

5. Indicate how the following quantities will transform under the P (space inversion) and T (time reversal) operation: (a)Position coordinate (b)Momentum vector (c)Spin or angular momentum vector $\vec{\sigma} = \vec{r} \times \vec{p}$ (d)Electric field $\vec{E} = -\nabla\phi$ (e)Magnetic field $\vec{B} = \vec{i} \times \vec{r}$

6. The deuteron is a bound state of neutron and proton and has spin 1 and positive parity. Prove that it can exist only in the $1S$ and $1D$ states.

7. The Δ^0 and Λ^0 both decay to proton and π^- meson. Explain why the Δ^0 meson lifetime is $\approx 10^-23s$ while that of Λ^0 is $2.6 \times 10^{-10}s$.

8. Conventionally nucleon is given positive parity. What does one say about deuteron's parity and the intrinsic parities of u and d-quarks?
   Answer: Deuteron consists of proton and neutron. Parity P satisfies the relation $P(|deuteron\rangle) = P(|n\rangle)P(|p\rangle)$. Nucleon has positive parity. So $P(|deuteron\rangle) = (+1)(+1) = +1$, positive parity. Nucleons consists of three quarks: $n = udd$, $p = uud$. The intrinsic parity of up quark must be positive so that $P(|n\rangle) = P(|u\rangle)P(|d\rangle)P(|d\rangle) = +1$ even if $P(|d\rangle)$ is negative. Similarly, the intrinsic parity of down quark must be positive so that $P(|p\rangle) = P(|u\rangle)P(|u\rangle)P(|d\rangle) = +1$.

9. Show that only states for which all the additive quantum numbers are zeros can be eigenstates of charge conjugation operator.
   Answer: Let an operator $\hat{C}$ changes a particle into its antiparticle. It is a

unitary operator. $\hat{C}\,|q\rangle = |-q\rangle$. In general $|-q\rangle \neq -\,|q\rangle$ because q is a function of additive quantum numbers component of isospin I_3, strangeness S and baryon number A. If $\hat{Q}$ is charge operator $\hat{Q}\,|q\rangle = q\,|q\rangle$ so that $|q\rangle$ is an eigenstate of $\hat{Q}$.

$$\hat{C}\hat{Q}\,|q\rangle = \hat{C}q\,|q\rangle = q\,|-q\rangle$$

$$\hat{Q}\hat{C}\,|q\rangle = \hat{Q}\,|-q\rangle = -q\,|-q\rangle$$
$$[\hat{C}\hat{Q} + \hat{Q}\hat{C}]\,|q\rangle = 0$$

That is, the anticommutator $[\hat{C},\hat{Q}]_+$ vanishes. $\hat{C}$ and $\hat{Q}$ do not commute. They cannot have simultaneous eigenstates. Hence $|q\rangle$ is not an eigenstate of $\hat{C}$. $|q\rangle \equiv |I_3, S, A\rangle$ will be an eigenstate of $\hat{C}$ if $|I_3, S, A\rangle = |-I_3, -S, -A\rangle \equiv |-q\rangle$ which implies $I_3 = S = A = L = 0$. Only states with $Q = 0, B = 0, S = 0, 1_3 = 0$ can be eigenstates of $\hat{C}$

10. Write down the isospin operators and eigenstates for the nucleon doublet. Use them to show how a proton can be transformed into a neutron and vice versa.
Answer: Like spin angular momentum, the symmetry group of isospin is $SU(2)$. The fundamental representation of $SU(2)$ is two-dimensional. The generators of this group are the traceless hermitian Pauli matrices. If I represents isospin with components I_1, I_2, I_3), its form will be $I_1 = \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $I_2 = \frac{1}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$, $I_3 = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. The two orthogonal eigenvectors in this space may be chosen as $\begin{pmatrix}1\\0\end{pmatrix} = |p\rangle$ and $\begin{pmatrix}0\\1\end{pmatrix} = |n\rangle$. Hence eigenvalues of I_3 are

$$\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}$$

A proton state can be changed to a neutron state by applying a transformation. Let $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$. That is $a = 0, c = 1$, b and d may be any number. Let $b = 1, d = 0$. Then $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$, $I_1\,|p\rangle = |n\rangle$.
If $b = 0, d = 0$

$$\frac{1}{2}\left[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} - i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\right]\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$
$$(I_1 - iI_2)\,|p\rangle = |n\rangle\,,\ I_-\,|p\rangle = |n\rangle$$

Glossary of terms used in high energy physics

1. annihilation:- A process in which a particle meets its corresponding antiparticle and both disappear The energy appears in some other form, perhaps as a different particle and its antiparticle (and their energy), perhaps as many mesons, perhaps as a single neutral boson such as a Z boson. The produced particles may be any combination allowed by conservation of energy and momentum and of all the charge types and other rules.

2. antimatter:- Material made from antifermions. We define the fermions that are common in our universe as matter and their antiparticles as antimatter. In the particle theory there is no a priori distinction between matter and antimatter. The asymmetry of the universe between these two classes of particles is a deep puzzle for which we are not yet completely sure of an explanation .

3. antiparticle:- For every fermion type there is another fermion type that has exactly the same mass but the opposite value of all other charges (quantum numbers). This is called the antiparticle. For example, the antiparticle of an electron is a particle of positive electric charge called the positron. Bosons also have antiparticles, except for those that have zero value for all charges, for example, a photon or a composite boson made from a quark and its corresponding antiquark. In this case there is no distinction between the particle and the antiparticle, they are the same object .

4. antiquark:- The antiparticle of a quark. An antiquark is denoted by putting a bar over the corresponding quark .

5. baryon:- A hadron made from three quarks. The proton (uud) and the neutron (udd) are both baryons. They may also contain additional quark-antiquark pairs .

6. baryon-antibaryon asymmetry:- The observation that the universe contains many baryons but few antibaryons; a fact that needs explanation .

7. boson:- A particle that has integer intrinsic angular momentum (spin) measured in units of $\hbar$ (spin = 0, 1, 2,...). All particles are either fermions or bosons. The particles associated with all the fundamental interactions are bosons. Composite particles with even numbers of fermion constituents (quarks) are also bosons .

8. bottom quark (b):- The fifth flavor of quark (in order of increasing mass), with electric charge -1/3 .

9. charge:- A quantum number carried by a particle. Determines whether the particle can participate in an interaction process. A particle with electric charge has electrical interactions; one with strong charge has strong interactions, etc .

10. charm quark (c):- The fourth flavor of quark (in order of increasing mass), with electric charge +2/3 .

11. color charge:- The quantum number that determines participation in strong interactions, quarks and gluons carry non-zero color charges .

12. color neutral:- An object with no net color charge. For composites made of color charged particles the rules of neutralization are complex. Three quarks (baryon) or a quark plus an antiquark (meson) can both form color-neutral combinations .

13. confinement:- The property of the strong interactions that quarks or gluons are never found separately but only inside color-neutral composite objects .

14. conservation:- When a quantity (e.g.- electric charge, energy or momentum) is conserved, it is the same after a reaction between particles as it was before .

15. decay:- A process in which a particle disappears and in its place two or more different particles appear. The sum of the masses of the produced particles is always less than the mass of the original particle .

16. down quark (d):- The second flavor of quark (in order of increasing mass), with electric charge -1/3 .

17. electric charge:- The quantum number that determines participation in electromagnetic interactions .

18. electromagnetic interaction:- The interaction due to electric charge; this includes magnetic effects which have to do with moving electric charges .

19. electroweak interaction:- In the Standard Model, electromagnetic and weak interactions are unified. Physicists use the term electroweak to encompass both of them .

20. fermion:- Any particle that has odd-half-integer (1/2, 3/2, ...) intrinsic angular momentum (spin), measured in units of $\hbar$. As a consequence of this peculiar angular momentum, fermions obey a rule called the Pauli Exclusion Principle, which states that no two fermions can exist in the same state at the same place and time. Many of the properties of ordinary matter arise because of this rule. Electrons, protons and neutrons are all fermions, as are all the fundamental matter particles, both quarks and leptons .

21. fixed-target experiment:- An experiment in which the beam of particles from an accelerator is directed at a stationary (or nearly stationary) target. The target may be a solid, a tank containing liquid or gas, or a gas jet .

22. flavor:- The name used for the different quark types (up, down, strange, charm, bottom, top) and for the different lepton types (electron, muon, tau). For each charged lepton flavor there is a corresponding neutrino flavor. In other words, flavor is the quantum number that distinguishes the different quark/lepton types. Each flavor of quark and charged lepton has a different mass. For neutrinos we do not yet know if they have a mass or what the masses are .

23. fundamental interaction:- In the Standard Model the fundamental interactions are the strong, electromagnetic, weak and gravitational interactions. There is at least one more fundamental interaction in the theory that is responsible for fundamental particle masses. Five interaction types are all that are needed to explain all observed physical phenomena .

24. fundamental particle:- A particle with no internal substructure. In the Standard Model the quarks, leptons, photons, gluons, $W^{\pm}$ bosons, and Z bosons are fundamental. All other objects are made from these .

25. generation:- A set of one of each charge type of quark and lepton, grouped by mass. The first generation contains the up and down quarks, the electron and the electron neutrino .

26. gluon (g):- The carrier particle of quark-quark interactions .

27. grand unified theory:- Any of a class of theories which contain the Standard Model but go beyond it to predict further types of interactions mediated by particles with masses of order $1015 GeV/c^2$. At energies large compared to this mass, the strong, electromagnetic and weak interactions are seen as different aspects of one unified interaction.

28. gravitational interaction:- The interaction of particles due to their mass-energy.

29. graviton:- The carrier particle of the gravitational interactions; not yet directly observed.

30. hadron:- A particle made of strongly-interacting constituents (quarks and/or gluons). These include the mesons and baryons. Such particles participate in residual strong interactions.

31. Higgs boson:- The carrier particle or quantum excitation of the additional force needed to introduce particle masses in the Standard Model. Not yet observed.

32. interaction:- It refers to exchange of momentum and energy between two systems.Also it is any process in which a particle decays or it responds to a force due to the presence of another particle as in a collision.

33. kaon (K):- A meson containing a strange quark and an anti-up (or an anti-down) quark, or an anti-strange quark and an up (or down) quark.

34. lepton:- A fundamental fermion that does not participate in strong interactions. The electrically-charged leptons are the electron (e), the muon (), the tau (), and their antiparticles. Electrically-neutral leptons are called neutrinos ().

35. LHC:- The Large Hadron Collider at the CERN laboratory in Geneva, Switzerland. LHC will collide protons into protons at a center-of-mass energy of about 14 TeV. When completed in the year 2005, it will be the most powerful particle accelerator in the world. It is hoped that it will unlock many of the remaining secrets of particle physics.

36. linac:- An abbreviation for linear accelerator, that is an accelerator that has no bends in it.

37. luminosity:- The number of particles per square-centimeter per second generated in the beams of high energy particle experiments. The higher the luminosity, the greater the number of events produced for study.

38. meson:- A hadron made from an even number of quark constituents. The basic structure of most mesons is one quark and one antiquark.

39. muon ():- The second flavor of charged lepton (in order of increasing mass), with electric charge -1.

40. muon chamber:- The outer layers of a particle detector capable of registering tracks of charged particles. Except for the chargeless neutrinos, only muons reach this layer from the collision point.

41. neutrino :- A lepton with no electric charge. Neutrinos participate only in weak and gravitational interactions and therefore are very difficult to detect. There are three known types of neutrino all of which are very light and could possibly even have zero mass.

42. neutron :- A baryon with electric charge zero; it is a fermion with a basic structure of two down quarks and one up quark (held together by gluons). The neutral component of an atomic nucleus is made from neutrons. Different isotopes of the same element are distinguished by having different numbers of neutrons in their nucleus.

43. nucleon:- A proton or a neutron; that is, one of the particles that makes up a nucleus.

44. photon:- The carrier particle of electromagnetic interactions.

45. pion :- The least massive type of meson, pions can have electric charges ± 1 or 0.

46. plasma:- A gas of charged particles.

47. positron :- The antiparticle of the electron.

48. proton (p):- The most common hadron, a baryon with electric charge (+1) equal and opposite to that of the electron (-1). Protons have a basic structure of two up quarks and one down quark (bound together by gluons). The nucleus of a hydrogen atom is a proton. A nucleus with electric charge Z contains Z protons; therefore the number of protons is what distinguishes the different chemical elements.

49. quark (q):- A fundamental fermion that has strong interactions. Quarks have electric charge of either 2/3 (up, charm, top) or -1/3 (down, strange, bottom) in units where the proton charge is 1.

50. residual interaction:- Interaction between objects that do not carry a charge but do contain constituents that have charge. Although some chemical substances involve electrically-charged ions, much of chemistry is due to residual electromagnetic interactions between electrically-neutral atoms. The residual strong interaction between protons and neutrons, due to the strong charges of their quark constituents, is responsible for the binding of the nucleus.

51. spin:- Intrinsic angular momentum of a particle, given in units of $\hbar$, the quantum unit of angular momentum, where $\hbar = 1.054x10 - 34Js$.

52. Standard Model:- Physicists' name for the theory of fundamental particles and their interactions. It is widely tested and is accepted as correct by particle physicists.

53. strange quark (s):- The third flavor of quark (in order of increasing mass), with electric charge -1/3.

54. strong interaction:- The interaction responsible for binding quarks, antiquarks, and gluons to make hadrons. Residual strong interactions provide the nuclear binding force.

55. synchrotron:- A type of circular accelerator in which the particles travel in synchronized bunches at fixed radius.

56. tau lepton:- The third flavor of charged lepton (in order of increasing mass), with electric charge -1.

57. top quark:- The sixth flavor of quark (in order of increasing mass), with electric charge 2/3. Its mass is much greater than any other quark or lepton.

58. up quark:- The least massive flavor of quark, with electric charge 2/3.

59. vertex detector:- A detector placed very close to the collision point in a colliding beam experiment so that tracks coming from the decay of a short-lived particle produced in the collision can be accurately reconstructed and seen to emerge from a 'vertex' point that is different from the collision point.

60. virtual particle:- A particle that exists only for an extremely brief instant as an intermediary in a process. The intermediate or virtual particle stages of a process cannot be directly observed. If they were observed, we might think that conservation of energy was violated. However, the Heisenberg Uncertainty Principle (which can be written as E . t $>$ /2) allows an apparent violation of the conservation of energy. If one sees only the initial decaying particle (such as a meson with the c quark) and the final decay products (such as s $+$ e $+$ e$+$), one observes that energy is conserved. The 'virtual' particle (such as the $W^{\pm}$) exists for such a short time that it can never be observed.

61. $W^{\pm}$ boson:- A carrier particle of the weak interactions. It is involved in all electric-charge-changing weak processes.

62. weak interaction:- The interaction responsible for all processes in which flavor changes, hence for the instability of heavy quarks and leptons, and particles that contain them. Weak interactions that do not change flavor (or charge) have also been observed..

63. Z-boson:- A carrier particle of weak interactions. It is involved in all weak processes that do not change flavor.

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